

QCD Theory

Part 3

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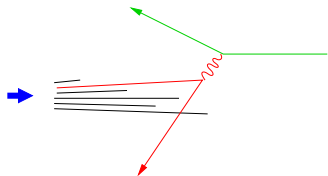
DESY Summer Student Programme 2017, Hamburg



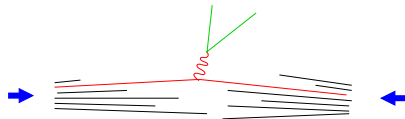
The parton model

description for deep inelastic scattering, Drell-Yan process, etc.

- ▶ fast-moving hadron
 \approx set of free partons (q, \bar{q}, g) with low transverse momenta
- ▶ physical cross section
 $=$ cross section for partonic process $(\gamma^* q \rightarrow q, q\bar{q} \rightarrow \gamma^*)$
 \times parton densities



Deep inelastic scattering (DIS): $l p \rightarrow l X$



Drell-Yan: $pp \rightarrow l^+ l^- X$



Nobel prize 1980 for
Friedman, Kendall, Taylor

The parton model

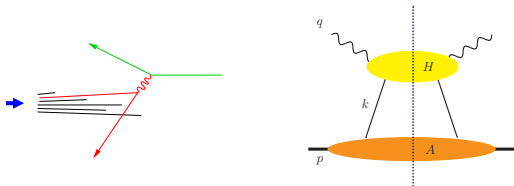
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Factorisation

- ▶ implement **and correct** parton-model ideas in QCD
 - conditions and limitations of validity
 kinematics, processes, observables
 - corrections: partons interact
 α_s **small at large scales** \rightsquigarrow **perturbation theory**
 - definition of parton densities in QCD
 derive their general properties
 make contact with non-perturbative methods

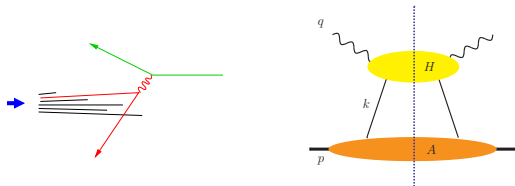
Factorisation: physics idea and technical implementation



- ▶ idea: separation of physics at different scales
 - high scales: quark-gluon interactions
 \rightsquigarrow compute in perturbation theory
 - low scale: proton \rightarrow quarks, antiquarks, gluons
 \rightsquigarrow parton densities

requires hard momentum scale in process
large photon virtuality $Q^2 = -q^2$ in DIS

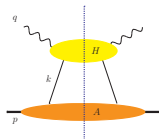
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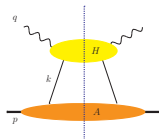
- ▶ implementation: separate process into
 - “hard” subgraph H with particles far off-shell
compute in perturbation theory
 - “collinear” subgraph A with particles moving along proton
turn into definition of parton density

Collinear expansion

- ▶ graph gives $\int d^4k H(k)A(k)$; simplify further
- ▶ light-cone coordinates \rightsquigarrow blackboard



Collinear expansion



- ▶ graph gives $\int d^4k H(k)A(k)$; simplify further
- ▶ in hard graph neglect small components of external lines
 \rightsquigarrow Taylor expansion

$$H(k^+, k^-, k_T) = H(k^+, 0, 0) + \text{corrections}$$

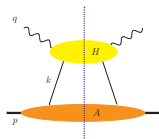
\rightsquigarrow loop integration greatly simplifies:

$$\int d^4k H(k) A(k) \approx \int dk^+ H(k^+, 0, 0) \int dk^- d^2k_T A(k^+, k^-, k_T)$$

- ▶ in **hard scattering** treat incoming/outgoing partons as exactly collinear ($k_T = 0$) and on-shell ($k^- = 0$)
- ▶ in collin. matrix element **integrate** over k_T and virtuality
 \rightsquigarrow collinear (or k_T integrated) parton densities
 only depend on $k^+ = xp^+$

further subtleties related with spin of partons, not discussed here

Definition of parton distributions



- ▶ matrix elements of quark/gluon operators

$$f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \frac{1}{2} \gamma^+ \psi(z) | p \rangle \Big|_{z^+=0, z_T=0}$$

$\psi(z)$ = quark field operator: annihilates quark

$\bar{\psi}(0)$ = conjugate field operator: creates quark

$\frac{1}{2} \gamma^+$ sums over quark spin

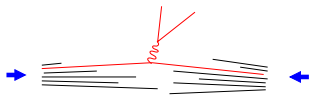
$\int \frac{dz^-}{2\pi} e^{ixp^+z^-}$ projects on quarks with $k^+ = xp^+$

- ▶ analogous definitions for polarised quarks, antiquarks, gluons
- ▶ analysis of factorisation used Feynman graphs but here provide **non-perturbative** definition

further subtleties related with choice of gauge, not discussed here

Factorisation for pp collisions

- ▶ example: Drell-Yan process $pp \rightarrow \gamma^* + X \rightarrow \mu^+ \mu^- + X$
where X = any number of hadrons
- ▶ one parton distribution for each proton \times hard scattering
 \rightsquigarrow **deceptively** simple physical picture



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 \rightsquigarrow **deceptively** simple physical picture



- ▶ “spectator” interactions produce additional particles which are also part of unobserved system X (“underlying event”)
- ▶ need not calculate this thanks to **unitarity** as long as cross section/observable **sufficiently inclusive**
- ▶ but must calculate/model if want more detail on the final state

More complicated final states

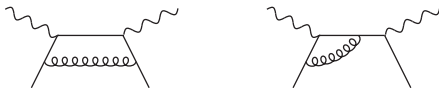
- ▶ production of W , Z or other colourless particle (Higgs, etc) same treatment as Drell-Yan
- ▶ jet production in ep or pp : hard scale provided by p_T
- ▶ heavy quark production: hard scale is m_c , m_b , m_t

Importance of factorisation concept

- ▶ describe high-energy processes: study electroweak physics, search for new particles, e.g.
 - discovery of top quark at Tevatron ($p + \bar{p}$ at $\sqrt{s} = 1.8 \text{ TeV}$)
 - measurement of W mass at Tevatron and LHC
 - determination of Higgs boson properties at LHC
- ▶ determine parton densities as a tool to make predictions and to learn about **proton structure**
 - require many processes to disentangle quark flavors and gluons

A closer look at one-loop corrections

- ▶ example: DIS

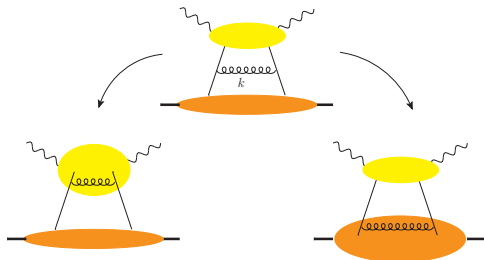


- ▶ UV divergences removed by standard renormalisation
- ▶ soft divergences cancel in sum over graphs
- ▶ collinear div. do **not** cancel, have integrals

$$\int_0^1 \frac{dk_T^2}{k_T^2}$$

what went wrong?

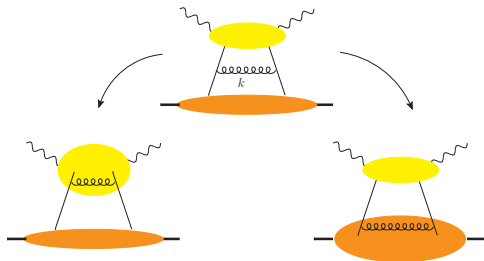
- ▶ hard graph should not contain internal collinear lines
collinear graph should not contain hard lines
- ▶ must not double count \rightsquigarrow factorisation scale μ



- ▶ with cutoff: take $k_T > \mu$
 $1/\mu \sim$ transverse resolution

take $k_T < \mu$

- ▶ hard graph should not contain internal collinear lines
collinear graph should not contain hard lines
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- ▶ with cutoff: take $k_T > \mu$
 $1/\mu \sim$ transverse resolution
- ▶ in dim. reg.:
subtract collinear divergence

take $k_T < \mu$
subtract ultraviolet div.

The evolution equations

► DGLAP equations

$$\frac{d}{d \log \mu^2} f(x, \mu) = \int_x^1 \frac{dx'}{x'} P\left(\frac{x}{x'}\right) f(x', \mu) = (P \otimes f(\mu))(x)$$

► $P =$ splitting functions



- have perturbative expansion

$$P(x) = \alpha_s(\mu) P^{(0)}(x) + \alpha_s^2(\mu) P^{(1)}(x) + \alpha_s^3(\mu) P^{(2)}(x) \dots$$

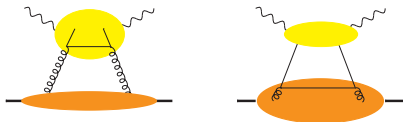
known to 3 loops

Moch, Vermaseren, Vogt 2004

- contains terms $\propto \delta(1-x)$ from virtual corrections



- ▶ quark and gluon densities mix under evolution:



- ▶ matrix evolution equation

$$\frac{d}{d \log \mu^2} f_i(x, \mu) = \sum_{j=q, \bar{q}, g} (P_{ij} \otimes f_j(\mu))(x) \quad (i, j = q, \bar{q}, g)$$



- ▶ parton content of proton depends on resolution scale μ

Factorisation formula

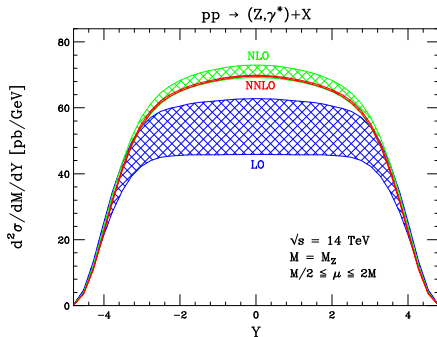
- ▶ example: $p + p \rightarrow H + X$

$$\sigma(p + p \rightarrow H + X) = \sum_{i,j=q,\bar{q},g} \int dx_i dx_j f_i(x_i, \mu_F) f_j(x_j, \mu_F) \\ \times \hat{\sigma}_{ij}(x_i, x_j, \alpha_s(\mu_R), \mu_R, \mu_F, m_H) + \mathcal{O}\left(\frac{\Lambda^2}{m_H^4}\right)$$

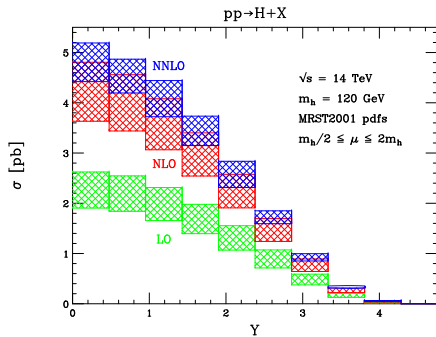
- $\hat{\sigma}_{ij}$ = cross section for hard scattering $i + j \rightarrow H + X$
 m_H provides hard scale
 - μ_R = renormalisation scale, μ_F = factorisation scale
may take different or equal
 - μ_F dependence in C and in f cancels up to higher orders in α_s
similar discussion as for μ_R dependence
 - accuracy: α_s expansion and power corrections $\mathcal{O}(\Lambda^2/m_H^2)$
- ▶ can make σ and $\hat{\sigma}$ differential in kinematic variables, e.g. p_T of H

Scale dependence

examples: rapidity distributions in Z/γ^* and in Higgs production



Anastasiou, Dixon, Melnikov, Petriello, hep-ph/0312266

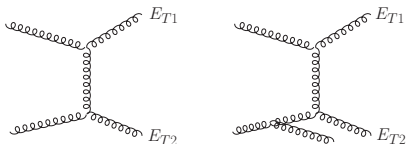


Anastasiou, Melnikov, Petriello, hep-ph/0501130

$$\mu_F = \mu_R = \mu \text{ varied within factor } 1/2 \text{ to } 2$$

LO, NLO, and higher

- ▶ instead of varying scale(s) may estimate higher orders by comparing N^n LO result with N^{n-1} LO
- ▶ caveat: comparison NLO vs. LO may not be representative for situation at higher orders often have especially large step from LO to NLO
 - ▶ certain types of contribution may first appear at NLO e.g. terms with gluon density $g(x)$ in DIS, $pp \rightarrow Z + X$, etc.
 - ▶ final state at LO may be too restrictive e.g. in $\frac{d\sigma}{dE_{T1} dE_{T2}}$ for dijet production



Summary of part 3

Factorisation

- ▶ implements ideas of parton model in QCD
 - perturbative corrections (NLO, NNLO, ...)
 - field theoretical def. of parton densities
↪ bridge to non-perturbative QCD
- ▶ valid for sufficiently inclusive observables
and up to power corrections in Λ/Q or $(\Lambda/Q)^2$
which are in general not calculable
- ▶ must in a consistent way
 - remove collinear kinematic region in hard scattering
 - remove hard kinematic region in parton densities
↔ UV renormalisation

procedure introduces factorisation scale μ_F

- separates “collinear” from “hard”, “object” from “probe”